national accelerator laboratory

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COEFFICIENTS OF POWER SERIES EXPANSION OF MAGNETIC LENGTH FROM MEASUREMENTS OF GRADIENT LENGTH

S. C. Snowdon

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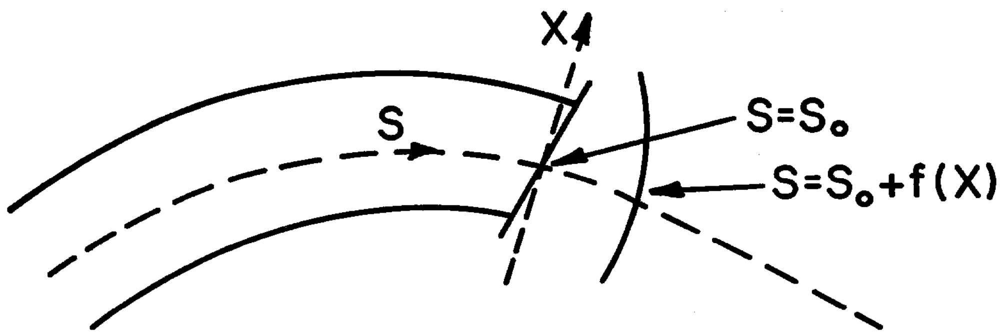
Let the effective azimuthal termination of the magnetic field within a magnet be described by a curve

$$s = s_0 + f(x) \tag{1}$$

where s is the usual variable measuring distance along the central orbit and x is the transverse distance orthogonal to s.



The meaning of an effective termination is that

$$B(x,s) = B(x)\{1-S(s-s_0-f(x))\}, \tag{2}$$

where S is a step function. Thus

$$B'(x,s) = B'(x)\{1-S\} + B(x)f'(x)\delta. \tag{3}$$

This gives on integrating from $s = s_1$, a position well within the magnet

$$\int_{s_1}^{\infty} B'ds = B'(x)(s-s_1) + B(x)f'(x). \tag{4}$$

Also

$$\int_{s_1}^{\infty} Bds = B(x)(s-s_1). \tag{5}$$

Eliminating $s-s_1$ between Eqs. (4) and (5) gives

$$f'(x) = \frac{B'(x)}{B(x)} \cdot \left\{ \frac{\int_{s_1}^{\infty} B'ds}{B'(x)} - \frac{\int_{s_1}^{\infty} BdS}{B(x)} \right\} . \tag{6}$$

But from Eq. (5)

$$\Delta L_B \equiv s-s_0 = \frac{\int_{s_1}^{\infty} Bds}{B(x)} - (s_0-s_1) \tag{7}$$

and from Eq. (1)

$$\Delta L_B = f(x) \tag{8}$$

Thus, if

$$\Delta L_G \equiv \frac{\int_{s_1}^{\infty} B'ds}{B'(x)} - (s_0-s_1) \tag{9}$$

it follows that

$$\boxed{\Delta L'_B = \frac{B'(x)}{B(x)} \cdot (\Delta L_G - \Delta L_B)} \tag{10}$$

PROBLEM

Given that

$$\Delta L_B = \Delta S + \sum_I D(I)x^I, \tag{11}$$

find ΔL_G.

It is convenient to represent B'(x) and B(x) in terms of dimensionless variables

$$g(x) = \frac{B'(x)}{B'(0)} \qquad b(x) = \frac{B(x)}{B(0)} . \tag{12}$$

To restore dimensions one introduces

$$k = \frac{B'(0)}{B(0)} , \tag{13}$$

the profile parameter. Thus Eq. (10) becomes

$$\Delta L_G - \Delta S - \sum_I D(I)x^I = \frac{1}{k} \cdot \frac{b(x)}{g(x)} \sum_I ID(I)x^{I-1}. \tag{14}$$

If one puts

$$T(J,I) = \frac{I}{k} \cdot \frac{b(x_J)}{g(x_J)} \cdot x_J{}^{I-1} + x_J{}^I, \tag{15}$$

and

$$GEND(J) = \Delta L(x_J) - \Delta S, \tag{16}$$

then

$$\sum T(J,I)D(I) = GEND(J), \tag{17}$$

where x is evaluated at the points x_J.

LEAST SQUARES ANALYSIS

Let

$$SUM(K) = \sum_J W(J)GEND(J)T(J,K), \tag{18}$$

and

$$C(K,L) = \sum_J W(J)T(J,K)T(J,L), \tag{19}$$

then

$$\sum_K C(K,L)D(K) = SUM(L). \tag{20}$$

Matrix inversion yields D(K).

The above considerations have been coded in the program GFITT.